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Research Report CCS 373

AN APPLICATION OF MATHEMATICAL
PROGRAMMING TO ASSESS MANAGERIAL
EFFICIENCY IN THE HOUSTON
INDEPENDENT SCHOOL DISTRICT

by

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ABSTRACT

→ A school may be viewed as an enterprise in which the professional staff provide the operating conditions for converting quantifiable resources or inputs into pupil learning (outputs). The resources are determined by budgets, teacher assignments, and student assignments while learning is determined by various outputs scored according to standardized tests such as the Iowa Test of Basic Skills. Following the work of Charnes, Cooper, and Rhodes [41], we use a ratio definition of efficiency which takes account of all outputs and inputs without requiring a priori specification of weights. Instead a series of mathematical programs are applied to determine "virtual multipliers" from actual data which yield the values that can be regarded as the "most favorable weights" for each school being evaluated. If the resulting optimum virtual multipliers for a given school yields an efficiency ratio of one, then that school is said to be efficient. If the ratio is less than one then that school is said to be inefficient relative to the other schools in the analysis. The ratio is also accorded operational significance--it is not merely an index number--so that the resulting values and the associated virtual multipliers make it possible to locate where improvements may be made along with their relative magnitudes.

This analysis was applied to 167 elementary schools in the Houston Independent School District. Of these schools, 78 were found to be inefficiently utilizing their resources as compared to the 89 efficient schools. Based on this pilot study, an Educational Productivity Council has been formed at the University of Texas at Austin to provide an annual analysis for all of its member schools. At present 285 Texas schools in 22 districts are scheduled for participation in the annual analysis as described in this investigation.

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I. INTRODUCTION

In the decade of the 1970's a pervasive push for accountability in public education in the United States [22] brought about revolutionary changes in the management problems faced by administrators of schools. These forces came to bear with swift and urgent impact on large metropolitan school districts such as the Houston Independent School District, a complex organization with 16,000 employees, 194,000 pupils, and an annual budget of \$320 million. Rapid population growth, federal regulatory agency mandates and court orders implementing civil rights legislation [15], combined with declining birth rates and alleged white flight to suburban schools, created a set of problems for which few management science techniques have been developed and tested. The one that the Board of Education felt was most important, and that provided the occasion for the research reported here, was the need for management information concerning the relative productivity of the 241 individual schools in relation to the resources allocated and the environmental factors affecting their outputs.

For five years, 1975-1979, the administration published an annual report of achievement scores for each school along with input data such as class size, professional staff characteristics, per pupil expenditures, ethnic group enrollment, and socioeconomic status [14]. However, no attempt was made to relate these inputs to the outputs of each school. Then in May 1979, in an effort to increase public confidence in Houston public schools,

the Board of Education published the Comparison of Academic Performance Data for Students in the Houston Independent School District and Other Public School Systems [29]. This was a bold and unprecedented step, but the report lacked a comprehensive model for making the comparisons required across the wide ranges of output and input mixes and levels involved, relying instead upon a simple comparison of mean achievement in schools, controlling for socio-economic level.

The need for a better quantitative model became imperative later that year when the state legislature adopted a law requiring statewide competency testing of pupils in all Texas schools and stipulated that "...student performance data shall be aggregated by campus and district and made available to the public, with appropriate interpretations..." [28].

1.1 Problem Description

Out of the context described above, the analysis on this study sought to provide a way to obtain a valid efficiency measure for each school that would be empirically based and logically justified and would also provide a measure of each school's ability to produce desired outputs from their valued inputs. It was also desired to compute the extent to which inputs were non-productive in inefficient school units and, if possible, to go even further and estimate the augmentations in outputs and/or the reductions in inputs that could be attained if efficiency were to be achieved.

Taken all together, then, a solution to this problem would provide management with information like the following:

- 1) An efficiency evaluation of individual schools which would include the productivity of the professional staff of the school while making allowance for the conditions under which they were operating.
- 2) Targeted output goals and identification of needed input modifications.
- 3) Identification of areas in which efficiency could be increased.

1.2 Deficiencies in Prior Methods Employed

We have been unable to find any methods, other than the one used in this investigation, that provide an overall operational definition of the efficiency of a school. There is, however, a considerable body of literature documenting attempts to define production functions in education. Levin [24] and Hanushek [16] characterize current methodologies as being deficient both conceptually and in their implementation. Following one such attempt, Levin [23] concluded, "The analysis...is fraught with difficulties that are unusually severe given the present analytical state of the art."

There are several difficulties which are immediately apparent in that the economic theory of production functions requires them to be extremal estimates whereas the statistical methods employed do not generally conform to this requirement.¹ In addition the situation of multiple outputs is not adequately dealt with.

The method most commonly employed is least squares linear regression with a single output. A variant method involves uses of multiple regressions--perhaps in repeated forms--to estimate the effects of the same inputs on different outputs. Some major criticisms of the methods that have previously been used to develop production functions may be summarized as:

- (i) Multiple outputs are not taken into account simultaneously. See Boardman, Davis and Sanday [5] who use econometric (simultaneous relation) estimates to circumvent this difficulty but do not deal with (or even discuss) the problem of obtaining extremal estimates.
- (ii) Regression coefficients do not necessarily indicate the most efficient way to produce an impact on outputs as noted by Bowles [7] who does not, however, provide any guidance on how this might be accomplished.
- (iii) There is a lack of agreement even on the meaning of economic terms such as "technical efficiency" vs. other kinds of efficiency when applied to schools (Levin [24]).
- (iv) The interdependency of inputs (multi-collinearity) and outputs, too, may produce misleading results if the resulting coefficients are used to determine alternatives in input mixes or levels (Bowles [7], Bowles and Levin [6]).

(v) The output-input relationships may not be linear or independent and, moreover, there are no guides available (theoretical or otherwise) for determining the classes of parametric functional forms to be used in these statistical estimation models (Levin [24], Bowles [7]).

(vi) None of the production function studies are based upon controlled experiments in which inputs have been manipulated (Averch et al. [2]). Note, however, that the theory of experimental design does not deal at all with problems involved in allowing for differences in managerial efficiency and/or other such variables that enter importantly into the resulting outputs.

The approach proposed by Charnes, Cooper and Rhodes [11,12] circumvents these difficulties. For one thing, it does not require the production functions to be specified in parametric form and, indeed, it allows for production functions which may differ for each school with multiple outputs and multiple inputs that may be related to each other in numerous ways (linear or nonlinear) that need not be specified. Furthermore, the resulting overall (scalar) measures of efficiency are obtained from extremal methods which relate the results to mathematical programming models in which all outputs are explicitly identified. The values assigned to these inputs and outputs, which are referred to as virtual

multipliers, help to locate sources of inefficiency on the one hand and also to indicate tradeoffs along the efficiency frontiers for additional use as required. How this is accomplished will be made clear, at least in part, in the sections that follows.

II. MATHEMATICAL MODEL

Following the notation of Charnes, Cooper, and Rhodes [12], we assume that we have observations on the same inputs and outputs for each of $j = 1, 2, \dots, n$ schools which we represent in the form

$$(1) \quad x_j = \begin{pmatrix} x_{1j} \\ x_{2j} \\ \vdots \\ x_{ij} \\ \vdots \\ x_{mj} \end{pmatrix}, \quad y_j = \begin{pmatrix} y_{1j} \\ y_{2j} \\ \vdots \\ y_{rj} \\ \vdots \\ y_{sj} \end{pmatrix}$$

where $x_{ij} > 0$ represents the observed value of the i th input for school j and $y_{rj} > 0$ represents the observed value of the r th output.

To determine the efficiency of any school k , say, from the set $j = 1, \dots, n$ we write

$$(2) \quad \begin{aligned} \max h_k &= \frac{\sum_{r=1}^s u_r y_{rk}}{\sum_{i=1}^m v_i x_{ik}} \\ \text{subject to} \quad 1 &\geq \frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}}, \quad j = 1, 2, \dots, n \\ 0 &< \epsilon \leq u_r, \quad r = 1, 2, \dots, s \\ 0 &< \epsilon \leq v_i, \quad i = 1, 2, \dots, m. \end{aligned}$$

Note that k appears in the constraints as well as the functional so that, automatically

$$(3) \text{ maximum } h_k = h_k^* \leq 1.$$

Moreover, all $y_{rj}, x_{ij} > 0$ with $u_r, v_i \geq \epsilon > 0$ guarantees that $h_k^* > 0$ will apply and a solution satisfying these conditions always exists.²

Drawing all of the above conditions together we then say that $h_k^* = 1$ if and only if school k is efficient relative to the other schools using these same inputs and producing these same outputs in the set of $j = 1, 2, \dots, n$ schools being considered.

It can be shown that this development can be related to the economic condition of Pareto optimality.³ Here, however, it will suffice to observe that we are according school k a favored position in that the optimal $u_r^*, v_i^* > 0$ are selected to give this school the highest possible efficiency rating that the data will allow. Hence, if $h_k^* < 1$ then this school is inefficient and cannot achieve a higher rating relative to the reference set with which it is being compared.

As we shall shortly see these $h_k^* < 1$ values can be accorded operational significance in the form of output increases or input decreases. For the present, however, we may observe that the above ratio form of the model involves a nonlinear programming problem. As Charnes, Cooper, and Rhodes have shown in [12], however, it may be replaced by an ordinary linear programming problem.

The latter development is accomplished by means of the theory of linear fractional programming as given in [9]. We do not repeat that development here, however, and merely replace the problem (2) by the following:⁴

$$\min g_k = \sum_{i=1}^m v_i x_{ik}$$

subject to

$$(3) \quad 0 \leq \sum_{i=1}^m v_i x_{ij} - \sum_{r=1}^s u_r y_{rj}, \quad j = 1, 2, \dots, n$$

$$1 = \sum_{r=1}^s u_r y_{rk}$$

$$v_i \leq v_i, \quad i = 1, 2, \dots, m$$

$$u_r \leq u_r, \quad r = 1, 2, \dots, s$$

We then call g_k^* = minimum g_k the reciprocal of the efficiency index defined by $h_k^* = \frac{1}{g_k^*}$.

Since the number of schools $j = 1, 2, \dots, n$ is usually much larger than the number of inputs and outputs to be considered, we take advantage of the duality relations of linear programming and replace (3) with the following:

$$\max z_k = \phi_k + \epsilon \sum_{r=1}^s s_r^+ + \epsilon \sum_{i=1}^m s_i^-$$

subject to

$$(4) \quad 0 = y_{rk} \phi_k - \sum_{j=1}^n y_{rj} \lambda_j + s_r^+, \quad r = 1, 2, \dots, s$$

$$x_{ik} = \sum_{j=1}^n x_{ij} \lambda_j + s_i^-, \quad i = 1, 2, \dots, m$$

$$\lambda_j, s_r^+, s_i^- \geq 0 \text{ for all } j, r, \text{ and } i.$$

At an optimum we have $z_k^* = g_k^*$ so that $h_k^* = \frac{1}{z_k^*}$ with the resulting relations showing how to move between the ratio and linear programming forms of the problems as required. There is now available extra information, however, in that any $s_r^{+*} > 0$ or $s_i^{-*} > 0$ also indicates that efficiency has not been achieved.⁵

Noting that $z_k^* > 1$ and/or $s_r^{+*} > 0$ or $s_i^{-*} > 0$ represent sources of inefficiency we next observe that efficiency can be attained if we apply these results to the original data in the form

$$(5) \quad \hat{x}_{ik} = x_{ik} - s_i^{-*}, \quad i = 1, 2, \dots, m$$

$$\hat{y}_{rk} = y_{rk} z_k^* + s_r^{+*}, \quad r = 1, 2, \dots, s.$$

In other words we adjust the original x_{ik} and y_{rk} observations in the manner indicated by (5) to obtain new values \hat{x}_{ik} , \hat{y}_{rk} which would render school k efficient. Note, in particular, that input reductions and output augmentations may be required simultaneously.

Computer software was written to solve n linear programming models defined in (4) in a single run via a new general purpose

linear programming code for the modified simplex method. For each school the program produces the optimal z_k^* , h_k^* , s_k^{*-} , s_k^{*+} , u_k^* , v_k^* , \hat{x}_k , \hat{y}_k and the results of ranging on the right-hand-side. The program begins with the initial feasible solution $\lambda_k = z_k = 1$ and all other variables zero. The LP code maintains the basis inverse in product form and only the non-zeros are stored. The LP code reinversion routine is based on the work of Hellerman and Rarick [19]. This routine incorporates a technique known as "splitting the bump" (see Kalan [20] and Orchard-Hays [25]) and uses the "spike swapping theory" of Helgason and Kennington [17]. A good description of these computational procedures may be found in Helgason [18]. The arrays and working files require approximately $(3n + 13)(s + m) + 4n$ words of core storage. This package for efficiency determination is described and documented in [21]. A revised package with more convenient data inputs and informational print out of solutions determines efficiency directly, instead of its reciprocal, and is presented in [1].

III. SELECTION OF OUTPUTS AND INPUTS

The Iowa Test of Basic Skills (ITBS), a comprehensive standardized test of achievement, had been administered to all elementary school pupils in Houston in May, 1978. Among other measures, the ITBS provided a composite score for the aggregated sub-test scores of each pupil; the mean of the composite score for grade three and the mean for grade six were used as the two output measures for each school.

This high level of aggregation -- over pupils and over sub-tests -- seemed adequate for the purpose of the study. If detailed individual school planning information were required, sub-test scores would be the relevant measures, with the scores aggregated by classroom. It should be noted also that available data restricted the analysis to cognitive outputs, although there are other school outputs that are valued.

The twelve input measures were selected from those that had been reported by the Houston administration in their annual Elementary School Profiles [14]. These included:

(A) *Measures of the characteristics of pupils that were highly correlated with achievement scores:*

(1 and 2) The previous year's achievement scores on the same test battery for the preceding grade levels -- 2nd and 5th grades,

- (3) percent non-minority enrollment,⁶
- (4) percent of students paying full lunch price, and
- (5) percent attendance: average daily attendance ÷ average daily membership

(B) *Measures of school resources available:*

- (6) number of professionals per 100 pupils,
- (7) local and state expenditures per pupil,
- (8) federal money allocated per pupil,
- (9) number of special programs in the school,
- (10) percent of teachers with masters degrees,
- (11) percent of teachers with more than 3 years experience, and
- (12) the percent of teaching days that the assigned teachers were present in the classroom: an average over all teachers assigned to the school; part-time teachers were prorated.

These were available measures regularly collected and understood⁷ by the school administration. They were aware that there were other desirable input measures, but chose to go ahead with the analysis to gain experience with the method which, if feasible and useful, would be repeated with more carefully designed measures.

IV. RESULTS

4.1 Output Tables and Interpretation

For the inputs and outputs described above, we calculated the inefficiency rating, z_k , for 167 elementary schools in the Houston Independent School District.⁸ This meant solving 167 linear programs each having 14 rows and 182 columns. Of these schools, 46.7% (78) were found to be inefficient. In order to display both efficiency and effectiveness, a graph relating efficiency and third grade achievement (see Figure 1) and a graph relating efficiency and sixth achievement (see Figure 2) were prepared. In order to present complete results and preserve the readability of the graphs, the following method was employed.

- 1) A school's number was used to indicate its associated point on the graph, thus providing identification of the school unit along with the measurement.
- 2) Inefficient schools that had the same efficiency rating and output measurement as some other school already on the graph were listed at the bottom of the figure, along with the number of the comparable school. For example, schools 25 and 73 both have $h_k = .91$ and third grade achievement of 3.5. School 25 is shown on the graph in Figure 1 and 73 is paired with 25 at the bottom of Figure 1.

Figures 1 and 2 About Here

Schools are commonly classified as above or below some achievement norm. However, it can be seen from Figures 1 and 2 that low achieving schools may be efficient. Consider efficient school 87. Given its very low outputs (2.9 and 5.2), it might seem that this school was not doing a good job. However, given its inputs, school 87 did as well as any school in the district; much better than inefficient school 49. Even though these two schools had very similar outputs, one was efficient and the other inefficient. From Table 1, it can be seen that school 49 did not fully utilize many of its inputs, the most notable being \$604/pupil of slack local and state funds. Thus appropriate planning and administrative response with respect to these two schools would be quite different.

Note that relative efficiency is displayed in Figures 1 and 2 (horizontal axis) along with normed measures of output (vertical axis). Thus, both effectiveness and efficiency may be considered together. No measures of absolute efficiency are provided, of course, but school administrators and board members are accustomed to measuring how well their schools perform on achievement tests relative to a norm. With the additional information provided by DEA, they are able to assess the relationship of these effectiveness indicators to the quantity of resources provided. Thus, a school that performs poorly with respect to achievement scores would be undesirable, even if efficient, but the management response would be different than it would be if the school were inefficient. This is discussed more fully in the following section.

Table 1 About Here

The information provided by DEA can be configured in many different ways. Since space is limited, it was decided simply to illustrate by including two possibilities⁹. Table 1 focuses on the most inefficient schools in the district. All these schools appear to have had adequate resources if fully utilized¹⁰. Table 2 highlights high-achieving, near-efficient schools. It might be that some reallocatable inputs from these schools could be transferred to efficient low-achieving schools. However, such reallocation decisions involve problems of relative evaluation and causal analyses that go beyond the scope of this paper. Hence the illustration that follow should be regarded as potential extensions that could flow naturally from a DEA analysis.

Table 2 About Here

4.2 Administrative Implications

The individual school is the production unit in a large school district. The General Superintendent and his central staff can increase productivity only indirectly -- through hiring and assignment of personnel and provision of resources and incentives that have the potential for increasing production if they are efficiently employed. As a result, information that the central administration may use to guide system planning is a crucial, and often missing, factor in top-level management of schools.

DEA provides this management information. The General Superintendent and his staff require school principals to submit annual operational plans

in which they set school goals and specify needed resources. These operational plans may employ DEA results as follows: (i) as information for school principals to use in their proposed plans, (ii) as management "audit" information for use in reviewing these plans before approval, (iii) as system scanning information to balance scarce resources among schools, and (iv) for annual review and evaluation of accomplishment of the previous year's plan.

In developing an operating plan for the individual school, the principal of an efficient school can increase output goals if achievement is below the norm and will be able to request additional resources for inputs that have opportunity costs. The principal of an inefficient school is able to identify those inputs that are unproductive. Furthermore, he can specify within-school plans for utilizing inputs more effectively in order to increase outputs to the level of expected achievement if the unit were efficient.

For example, a sample planning form for school 139 is illustrated in Display 1. Planning documents of this type have proved to be helpful with respect to communication. Using Display 1, the principal would observe that 2nd grade achievement in 1977 was at the norm (2.8), yet these same children a year later in 3rd grade were .5 below the norm. Further, there seems to be a cumulative loss in 4th, 5th, and 6th grades as 5th is 1.2 below the norm and 6th is 1.4. Thus, a concentrated effort to improve achievement in the intermediate grades is indicated; that is, the 6th grade needs to "catch up". School faculty should examine their special programs,

purchases with instructional dollars and the utilization of instructional personnel as the display indicates that these resources are not being effectively used.

Display 1 About Here

For management audit of proposed plans, the General Superintendent is able to use efficiency, inefficiency, slack, and opportunity cost information for such purposes as: determining whether low achievement schools are setting realistic improvement goals, whether additional input request are justified, and what inputs are not being fully utilized. An inefficient, low achieving school should be able to achieve outputs specified if efficient because other schools in the district with similar inputs were able to achieve such outputs. Consequently, requests for additional input would not seem to be justified. On the other hand an efficient, low achieving school could not reasonably be expected to increase its output unless the district provided more input as all resources were used as effectively as in any school in the district. Via the use of ranging information on the right-hand-side of the linear program, the superintendent can determine the amount of additional inputs which can be productively used under the current production rate. Without DEA results, such review cannot be done except in the most general way based upon average allocations and achievement norms.

As a means of balancing resources, the central staff can identify where slack resources exist in inefficient schools with high output. These slack resources might be better utilized in efficient schools with low output. For example, from Figures 1 and

2 we find that school 8 is efficient but low achieving (3rd grade = 3.0, 6th grade = 4.7) while school 63 is inefficient but high achieving (3rd grade = 4.7, 6th grade = 7.5). From Table 2, we see that school 63 is not productively utilizing .7 professionals per 100 pupils, \$326 per pupil of state and local funds, and 32.7% of their teachers with Masters Degree. Given this information, the General Superintendent might request (1) that the staff of School 8 submit plans for improving achievement along with specifications of additional input requirements needed to accomplish these goals and (2) the staff of school 63 to submit plans for effectively utilizing their non-productive resources. In reviewing these plans, some of the non-productive inputs from School 63 might be reallocated to School 8 with no expected loss in achievement at School 63 and an anticipated improvement at School 8. A subsequent analysis at the end of the next school year would allow the General Superintendent to assess the success of the plans. Of course, the General Superintendent would want to consider plans from all the schools, not just two. The above comparison of two schools is intended to illustrate how DEA results could be used for balancing resources.

At the completion of each year, use of DEA results can show whether goal achievement is as expected and whether input increments previously authorized were efficiently used. An overall audit of the system is possible to determine whether the proportion of efficient schools is increasing or decreasing.

Since the analysis compares each school to the observations obtained on all other schools, it follows that an element of competitiveness is involved. If an efficient school, through better operational plans succeeds in raising its achievement more than others, then some schools which were formerly efficient might become inefficient. Likewise, some inefficient schools might be reduced to even greater inefficiency. Competition is seen to be desirable by school administrators with whom we have worked so long as it does not unfairly penalize those schools which have many obstacles to overcome. Fairness of results is evidenced by the number of efficient schools in Figures 1 and 2 which have low achievement because of high proportions of children from disadvantaged families. We also found that faculties of schools who had been striving for better results were pleased to have an objective measure of their program.

V. SUMMARY AND CONCLUSION

The General Superintendent of Houston's school system found the pilot testing of DEA in 1979 to be sufficiently interesting so that he had the results distributed to area superintendents and to school principals. In addition, the District authorized a second analysis using 1980 results of the new mandatory statewide competency test of basic skills. This analysis will be used to gain additional experience with the methodology.

The validity of DEA was assessed in an informal test in which the General Superintendent and his staff first identified "trouble" schools and "outstanding" schools. In the two hour session approximately 40 of the 167 schools were reviewed in detail and a check of administrative assessment against DEA solutions resulted in 100% correct classification; i.e. the "trouble" schools were all inefficient to some degree and the outstanding schools were efficient. Equally important, the reasons for a school's status based on known local conditions generally coincided with the DEA slack values.

The three major problems we encountered in this pilot study were (1) obtaining data to specify adequate input measures, (2) obtaining data to specify outputs that were not limited to cognitive test results, and (3) difficulties in communicating the results of a complex quantitative process to those affected by the results. It is hoped that these problems will become less severe with increasing experience.

Other applications of DEA in Texas schools are currently underway.

A network of school districts has recently been formed with a membership of 235 elementary schools and 50 secondary schools in 22 Texas school districts. Known as the Educational Productivity Council, the schools contribute an annual membership fee to support an assessment similar to the one reported in this investigation. In addition, the member schools will cooperatively define the relevant inputs and outputs for their use and mutually share in efforts to develop better methods of application.

At another level of application, one might use the individual pupil as the unit of analysis, DEA can identify pupils making inefficient use of their resources for learning; this information can be aggregated at the classroom level to measure management efficiency of teachers in producing a learning environment.

This methodology is appropriate for use in non-school applications as well. For example, it is now being applied to highway safety units. Traffic violations and convictions are being used as outputs and the number and assignment of patrol units as inputs. Other potential uses for not-for-profit enterprises include the efficiency analysis of welfare units, emergency medical service units, and post office mail delivery.

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Table 1 Schools Less Than 90% Efficient

School		Outputs		Alterable Slack Resources (Inputs)							
h_k	k	3rd grade	6th grade	Profess per 100 pupils	Local + State \$/pupil	Fed \$ per pupil	Stu. Attendance (%)	Tea. Attendance (%)	Tea. with Masters Degrees (%)	Tea. with > 3 yrs. exp (%)	# Special Programs
.82	23	3.1	5.2	2.7	\$370	\$ 44	2.8%	6.1%	5.0%	8.6%	5.0
.83	12	3.3	5.7	0	\$ 24	\$ 10	0	5.3%	4.5%	17.7%	1.3
.83	30	3.1	5.3	1.4	\$ 28	\$ 93	4.0%	0	2.2%	15.8%	0
.84	49	3.0	5.1	4.0	\$604	\$107	1.9%	4.8%	0	0	.5
.85	72	3.1	5.5	.1	0	\$ 36	3.4%	2.2%	9.6%	0	1.3
.86	139	3.3	5.4	.5	\$ 60	\$ 82	0	1.5%	8.9%	18.8%	1.5
.86	155	3.2	5.8	0	\$ 34	0	0	3.2%	0	5.8%	0
.88	116	3.4	5.2	.1	\$ 74	0	6.4%	3.9%	23.4%	17.1%	3.1
.88	148	3.8	6.6	0	0	0	1.4%	0	2.1%	8.4%	.3
.89	138	3.5	6.4	0	\$145	\$ 16	0	.8%	0	0	0
.89	70	3.2	5.6	.8	0	0	4.9%	5.4%	0	0	.4
.89	44	3.1	5.2	2.0	\$250	\$176	4.0%	7.6%	0	25.6%	3.7
.89	165	3.8	6.5	0	0	0	.4%	0	4.5%	29.2%	0
.89	136	3.7	6.4	.6	\$ 70	\$ 35	1.5%	3.4%	11.5%	0	0

Table 2 High-Achieving Near-Efficient Schools

School		Outputs		Reallocatable Slack Resources (Inputs)			
h_k	k	3rd grade	6th grade	Profess/ 100 pupils	Local + State \$/pupil	Teachers with Masters Degrees (%)	Teachers with 3 years experience (%)
.94	11	4.3	7.2	.4	0	13.3%	28.8%
.95	158	4.3	7.5	0	\$ 50	0	23.2%
.96	100	4.4	7.2	1.3	\$326	18.0%	0
.96	133	4.5	7.3	.1	0	0	0
.96	119	4.5	7.5	0	\$ 99	20.1%	18.2%
.97	145	3.6	7.1	0	0	6.0%	0
.97	63	4.7	7.5	.7	\$326	32.7%	0
.98	52	4.3	7.3	0	\$ 93	5.3%	0
.98	154	4.5	7.5	0	\$ 33	11.5%	14.9%
.99	129	4.7	8.0	0	\$ 93	7.9%	18.6%

Display 1. Sample Planning Form

School 139

$h_{139} = .860$

Measurement Name	Observed Measurement	Measurement if Efficient	Measurement Goal
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Output

3rd Grade (78) (norm=3.8)	3.3 Grade Equiv	3.6	
6th Grade (78) (norm=6.8)	5.4 Grade Equiv	6.3	

Input

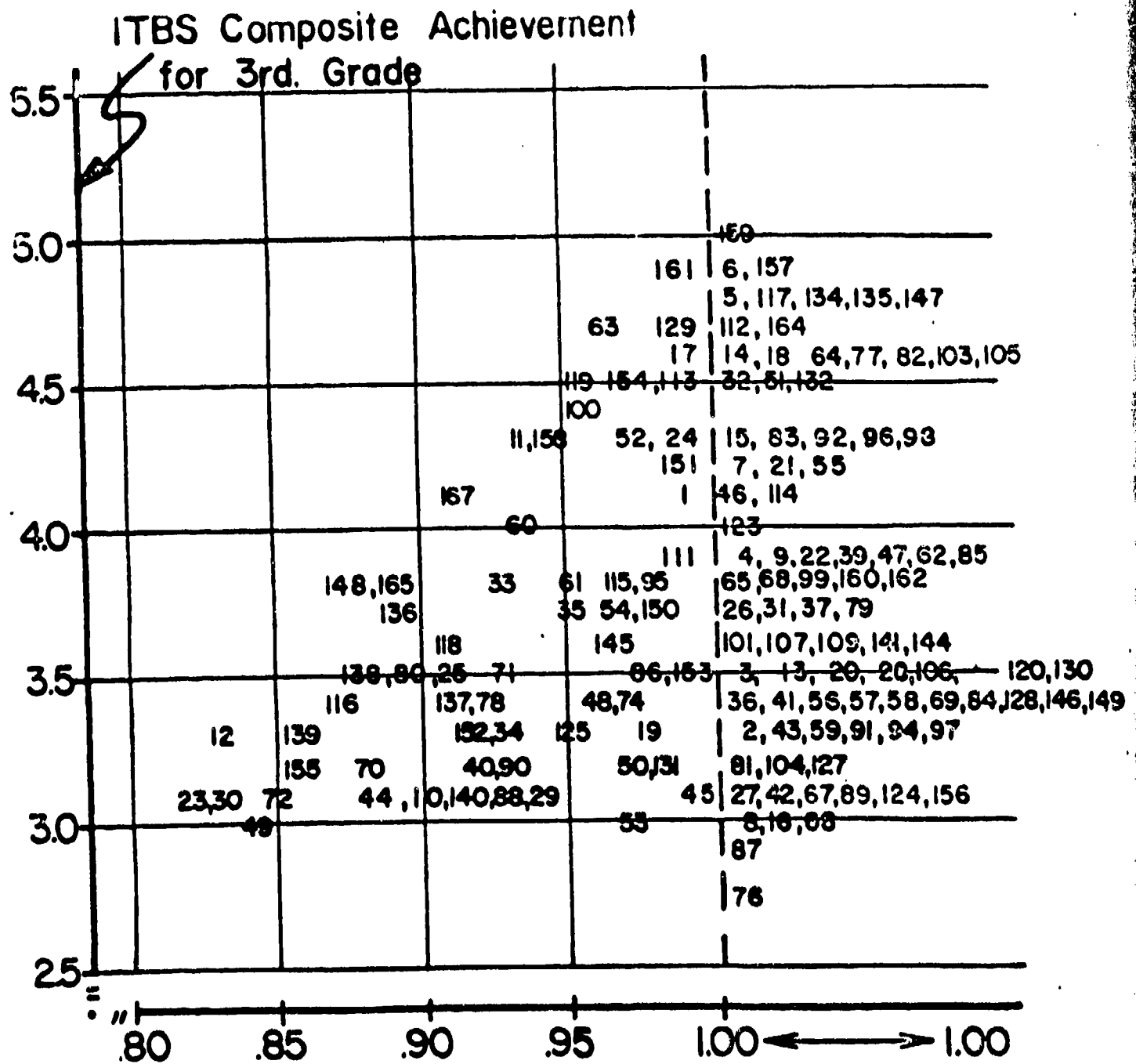
2nd Grade (77) (norm=2.8)	2.8 Grade Equiv	2.7	
5th Grade (77) (norm=5.8)	4.6 Grade Equiv	4.5	
% Non-minority	1.7 %	1.7	
% Pay lunch	22.0 %	22.0	
# Prof./100	5.9 prof/100 pupil	5.4	
Federal \$/pupil	\$210/pupil	\$128	
Local-State \$/pupil	\$1185/pupil	\$1125	
% Attendance	90.8 %	90.8	
% Teacher days worked	94.5 %	93.0	
% Masters degree	44.0 %	35.1	
% More 3 yrs. exp.	69.0 %	50.2	
# special programs	6 special programs	4.5	

Program Objectives

FOOTNOTES

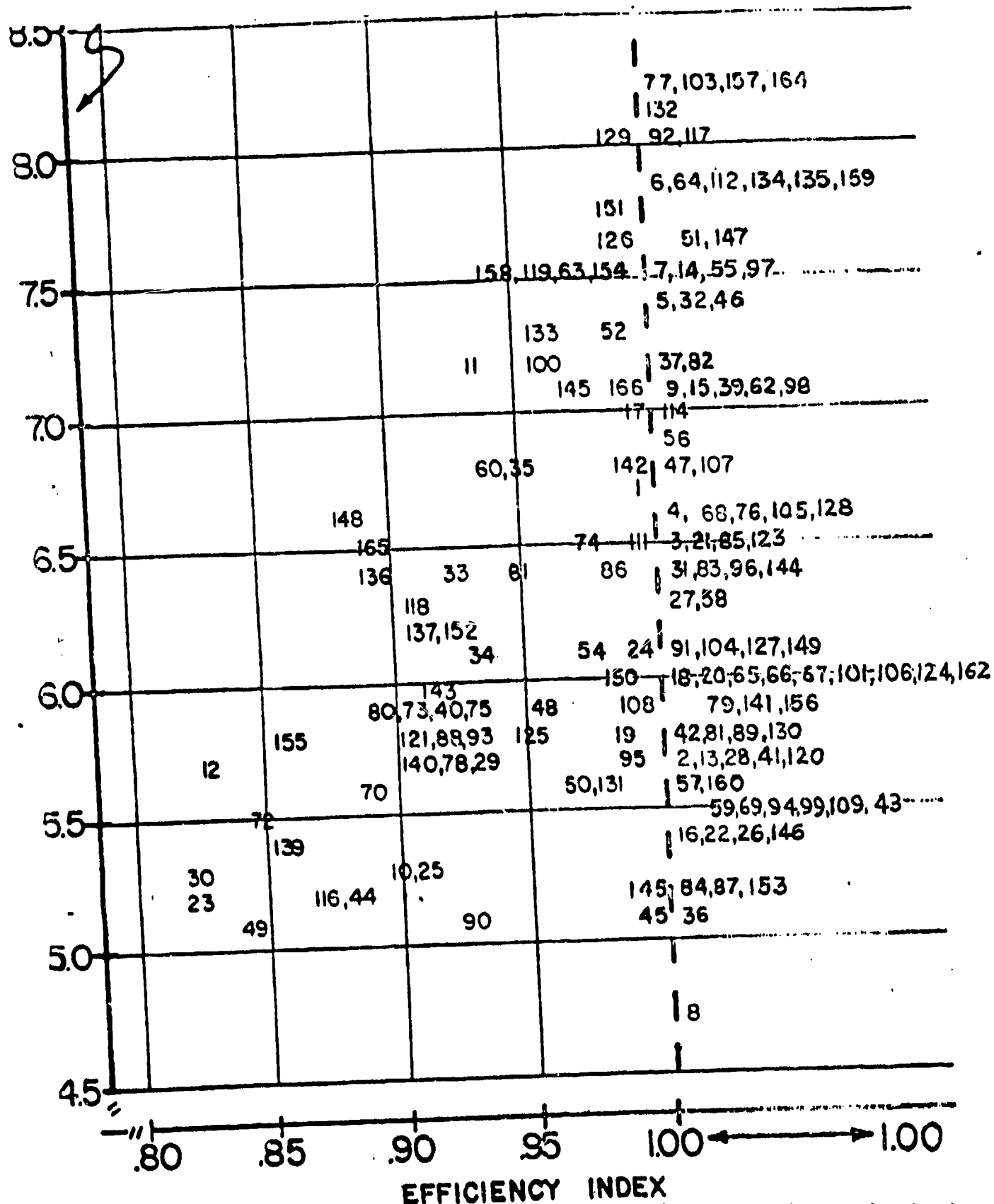
1. See in Levin [23] and Charnes, Cooper, Rhodes [11] for further discussion.
2. See Charnes, Cooper, Rhodes [12].
3. See Charnes, Cooper, Rhodes [12].
4. We also do not use the change of variables that is required to relate the two problems to each other but rely on the context to make clear the relations between the variables in (2) and (3).
5. The precise conditions for optimality and their relations to the non-Archimedean $\epsilon > 0$ values are set forth in detail in [10].
6. Note that inputs are defined in directionality so that increasing units of input would be expected to yield increasing units of output.
7. Note that percentages were used in scaling several inputs since in each case the measure was commonly employed in that form by administrators. As Sherman showed in [26], a use of ratios may produce misclassifications. This was checked, however, in a variety of ways. For instance, data were rescaled as log transforms and squared proportions and there were no significant differences in computed efficiency index values or in classification of units as efficient or inefficient. Since the ratio approach is common in educational management and since no adverse consequences were detected in the above described tests, we decided to continue with these ratios in the rest of the study.

8. Only elementary schools that had been in operation more than two years were included.
9. For other variants, see Bessent and Bessent [4].
10. Inputs such as minority group enrollment and socio-economic status are not shown in the tables since these are not alterable by management decision or intervention. They may affect the efficiency, however, and in actual practice would be displayed. They are deleted here to make the tables more readable.



EFFICIENCY INDEX																
Inefficient School not on Plot	38	73	75	93	102	108	110	121	122	126	133	142	143	163	165	
Same as School Numbered	125	25	33	90	88	153	61	80	11	129	119	1	20	44	113	

Fig. 1 Efficiency and Third Grade Achievement



Inefficient School not on Plot	38	53	71	102	110	113	115	122	138	161	163	167
Same as School Numbered	154	52	33	40	35	17	52	60	136	38	44	118

Fig. 2 Efficiency and Sixth Grade Achievement

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account of all outputs and inputs without requiring a priori specification of weights. Instead a series of mathematical programs are applied to determine "virtual multipliers" from actual data which yield the values that can be regarded as the "most favorable weights" for each school being evaluated. If the resulting optimum virtual multipliers for a given school yields an efficiency ratio of one, then that school is said to be efficient. If the ratio is less than one then that school is said to be inefficient relative to the other schools in the analysis. The ratio is also accorded operational significance--it is not merely an index number-- so that the resulting values and the associated virtual multipliers make it possible to locate where improvements may be made along with their relative magnitudes.

This analysis was applied to 167 elementary schools in the Houston Independent School District. Of these schools, 78 were found to be inefficiently utilizing their resources as compared to the 89 efficient schools. Based on the pilot study, an Educational Productivity Council has been formed at the University of Texas at Austin to provide an annual analysis for all of its member schools. At present 285 Texas schools in 22 districts are scheduled for participation in the annual analysis as described in this investigation.

Key Words Continued

Decision Making Units (DMU's)
Public Programs
School Achievement
Multiple Inputs
Multiple Outputs
Fractional Programming
Data Envelopment Analysis (DEA)